Solving Related Rates Problems

What has always distinguished calculus from algebra is its ability to deal with variables that change over time. It is quite easy to move from a formula relating static variables to a formula that relates their rates of change: Simply differentiate the formula implicitly **with respect to time**. This introduces an important category of problems called *related rates problems* that constitutes one of the most important applications of calculus.

Guidelines to solving related rate problems

1. Draw a picture.
2. Make a list of all known and unknown rates and quantities.
3. Relate the variables in an equation.
4. Differentiate with respect to time.
5. Substitute the known quantities and rates and solve.

IMPORTANT: Substituting a non-constant quantity before differentiating is not allowed!

Example 1: The length of a rectangle is decreasing by 2 inches per second and the width is increasing by 3 inches per second. When the length of a rectangle is 10 inches, and the width is 6 inches:

a) Determine the rate of change of the perimeter of the rectangle.	b) Determine the rate of change of the area of the rectangle.

Example 2: The radius of a circle is increasing at a rate of 4 cm/sec. At the moment when the radius is 20 cm:

a) How fast is the area of the circle changing?	b) How fast is the circumference of the circle changing?

Example 3: A cylindrical tank with a radius of 5 meters is being filled with water at a rate of 3 cubic meters per minute. How fast is the height of the water increasing?

Example 4: A police car, approaching a right-angled intersection from the north is chasing a speeding car that has turned the corner and is now moving straight east. When the police car is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with a radar gun that the distance between them and the car is increasing at 20 mph. If the police car is moving at 60 mph at the instant of measurement, what is the speed of the car?

Example 5: "The Ladder Problem" A 17-foot ladder is leaning against a wall. The ladder forms a right triangle with the wall and the ground. The base of the ladder slides away from the wall at a rate of 3 ft/sec. At the moment when the top of the ladder is 8 feet from the ground,

a) How fast is the top of the ladder sliding down the wall?
b) How fast is the angle formed by the ladder and the ground changing?
c) How fast is the area of the triangle changing?

Example 6: A spherical balloon is expanding at a rate of 60π in^3/sec. How fast is the surface area of the balloon expanding when the radius of the balloon is 4 inches? $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

Example 7: Water is leaking from a conical tank that is 20 feet in diameter and 50 feet deep. The rate at which water is leaking is 12 cubic feet per minute. How fast is the water level falling when the depth of the water in the reservoir is 20 feet?

Example 8: **“The Streetlight Problem”** A young boy is out at night, running toward a streetlamp at 6 ft/sec. If the streetlamp is 30 feet tall and the boy is 5 feet tall, how fast is the length of his shadow changing when he is 4 feet from the base of the lamppost?

Example 9 A balloon rises at a rate of 2 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 25 meters above the ground.